GRADE 12 DIPLOMA EXAMINATION

Mathematics 30

June 1984

Alberta
EDUCATION

GRADE 12 DIPLOMA EXAMINATION
MATHEMATICS 30

DESCRIPTION

Time: 2½ hours

Total possible marks: 65

This is a **CLOSED-BOOK** examination consisting of two parts:

PART A: 52 multiple-choice questions each with a value of 1 mark.

PART B: Five written-response questions for a total of 13 marks.

A mathematics data booklet is provided for your reference. Approved calculators may be used.

GENERAL INSTRUCTIONS

Fill in the information on the answer sheet as directed by the examiner.

For multiple-choice questions, read each carefully and decide which of the choices BEST completes the statement or answers the question. Locate that question number on the answer sheet and fill in the space that corresponds to your choice. Use an HB pencil only.

Example **Answer Sheet**

This examination is for the subject area of

A	B	C	D
○	○	○	●

A. Chemistry
B. Biology
C. Physics
D. Mathematics

If you wish to change an answer, please erase your first mark completely.

For written-response questions, read each carefully and write your answer in the space provided in the examination booklet.

DO NOT FOLD EITHER THE ANSWER SHEET OR THE EXAMINATION BOOKLET.

The presiding examiner will collect the answer sheet and examination booklet for transmission to Alberta Education.

JUNE 1984

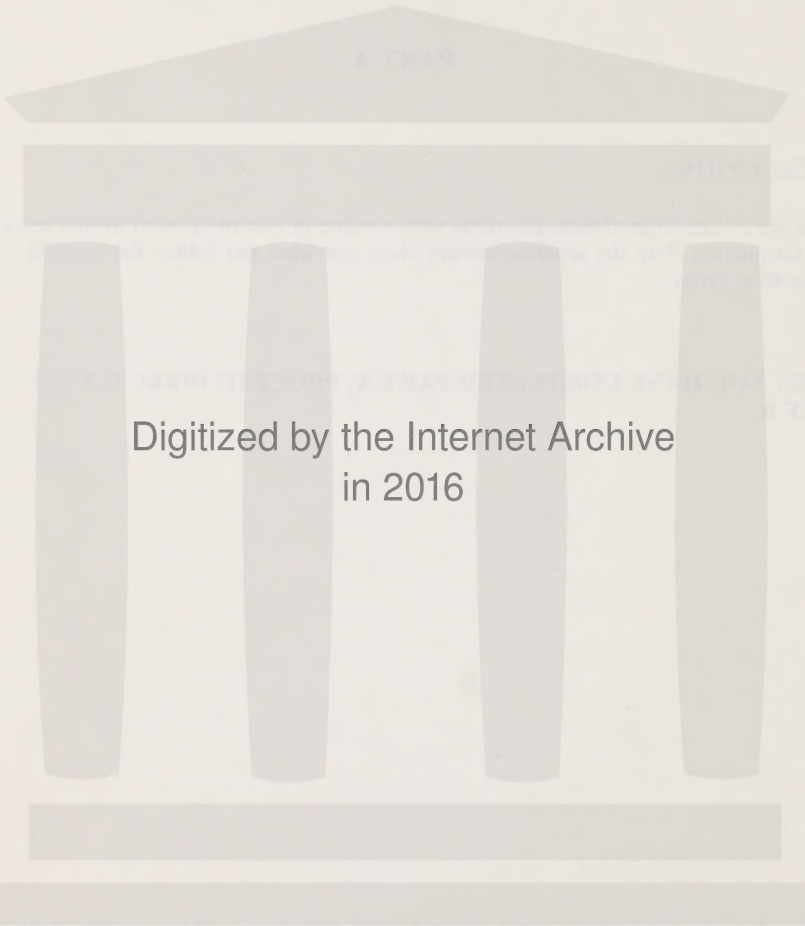
PART A

INSTRUCTIONS

There are 52 multiple-choice questions with a value of one mark each in this section of the examination. Use the separate answer sheet provided and follow the specific instructions given.

WHEN YOU HAVE COMPLETED PART A, PROCEED DIRECTLY TO PART B.

DO NOT TURN THE PAGE TO START THE EXAMINATION UNTIL TOLD TO DO SO BY THE PRESIDING EXAMINER.



1. A path length of $\frac{3\pi}{4}$ on a unit circle would cover

A. $\frac{3}{16}$ of the circle

B. $\frac{3}{8}$ of the circle

C. $\frac{3}{4}$ of the circle

D. $\frac{3}{2}$ of the circle

2. The exact value of $\csc\left(-\frac{2\pi}{3}\right)$ is

A. $-\frac{2}{\sqrt{3}}$

B. $-\frac{\sqrt{3}}{2}$

C. $-\frac{1}{2}$

D. -2

3. Expressed in radians, a measure of 235° equals

A. $-\frac{72\pi}{47}$

B. $\frac{47\pi}{72}$

C. $-\frac{36\pi}{47}$

D. $\frac{47\pi}{36}$

4. The solution set of $2 \sin^2 \theta - \sin \theta - 1 = 0$ where $0 \leq \theta < 2\pi$ is

A. $\left\{\frac{\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

B. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

C. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}\right\}$

D. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$

5. If $\sec \theta = -2$, then one value for $\tan \theta$ is

A. $-\frac{1}{\sqrt{3}}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

6. An equivalent form of $(\sin A + \cos A)^2 - 1$ is

A. $2 \sin A \cos A$

B. $2 \sin A \cos A - 2$

C. $\sin^2 A + \cos^2 A$

D. $\sin^2 A + \cos^2 A - 1$

7. $\tan(-15°)$ is equal to

A. $\sqrt{3} + 2$

B. $2 - \sqrt{3}$

C. $\sqrt{3} - 2$

D. $-\sqrt{3} - 2$

8. The domain of the cotangent of θ is

A. $\left\{ \frac{y}{x} \mid \frac{y}{x} \in R \right\}$

B. $\left\{ \frac{x}{y} \mid y, x \in R \right\}$

C. $\left\{ \theta \mid \theta = \frac{\pi}{2} \pm n\pi, n \in I \right\}$

D. $\left\{ \theta \mid \theta \in R, \theta \neq \pi \pm n\pi, n \in I \right\}$

9. The value of $\sin 195°$ is equal to

A. $-\sin 75°$
B. $\sin 75°$
C. $-\sin 15°$
D. $\sin 15°$

10. In obtuse $\triangle ABC$, if $\angle C = 30°$, $a = 3$, and $c = 4$, then $\angle A$, to the nearest degree, is

A. 60°
B. 68°
C. 37°
D. 22°

11. The area of the right-angled triangle ABC in terms of hypotenuse c and angle B is

A. $c^2 \sin B \cos B$

B. $2c \sin B \cos B$

C. $\frac{c \sin B \cos B}{2}$

D. $\frac{c^2 \sin B \cos B}{2}$

12. The graph of the relation defined by the equation $x^2 + y^2 - 6x + 4y - 12 = 0$ is a circle with centre at

A. $(-3, 2)$
B. $(3, -2)$
C. $(-2, 3)$
D. $(2, -3)$

13. The equation of a circle with centre $C(-1, 4)$ and tangent to the line $y = 0$ is

A. $(x + 1)^2 + (y - 4)^2 = 16$

B. $(x - 1)^2 + (y + 4)^2 = 16$

C. $(x + 1)^2 + (y - 4)^2 = 25$

D. $(x - 1)^2 + (y + 4)^2 = 25$

14. If the equation of a parabola is $36y = x^2$, then the focus and directrix respectively are

A. $F(0, 9)$ and $y = -9$
B. $F(0, -9)$ and $y = 9$
C. $F(9, 0)$ and $x = -9$
D. $F(-9, 0)$ and $x = 9$

15. The parabola with vertex $(0, 0)$ and focus $(3, 0)$ is defined by

A. $y^2 = -12x$

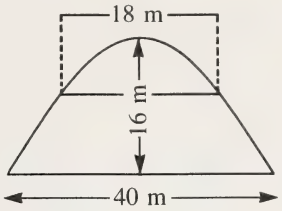
B. $y^2 = 12x$

C. $x^2 = -12y$

D. $x^2 = 12y$

16. A river 40 m wide is bridged by an arch in the form of a parabolic segment. Its altitude is 16 m. A reinforcing girder 18 m long is parallel to the base. The shortest distance from the girder to the vertex is

A. 3.24 m
B. 12.96 m
C. 6.48 m
D. 7.20 m



17. The length of the major axis of the ellipse defined by $4x^2 + 9y^2 = 36$ is

A. 4
B. 3
C. 9
D. 6

18. Given that $a > b$, the equation of an ellipse with centre $(0, 0)$ and one focus at $(c, 0)$ is

A. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

B. $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$

C. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

D. $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

19. The graph of $x^2 - 4y^2 - 3 = 0$ is

A. a hyperbola
B. an ellipse
C. a parabola
D. two straight lines

20. The equation of an asymptote to a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by

A. $y = \frac{b}{a^2}x$

B. $y = \frac{b^2}{2a}x$

C. $y = -\frac{b}{a}x$

D. $y = \frac{b^2}{a^2}x$

21. The length of the transverse axis of a hyperbola whose centre is at the origin is 24, and the length of the conjugate axis is 18. How far is the focus from the origin?

A. 13
B. 15
C. 21
D. 42

22. The equation of the hyperbola with centre at $(0, 0)$, one focus at $(5, 0)$, and one vertex at $(4, 0)$ is

A. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

B. $\frac{x^2}{16} - \frac{y^2}{25} = 1$

C. $\frac{x^2}{9} - \frac{y^2}{16} = -1$

D. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

23. Assuming that the series $2 + 5 + 8 + \ldots$ is arithmetic, the 23rd term is

A. 68
B. 71
C. 122
D. 137

24. Bill begins a savings plan by saving \$1.00 during the first week. In each subsequent week, he saves three dollars more than the week before. At the end of the 20th week, the total amount he has saved is

A. \$58
B. \$580
C. \$590
D. \$1160

25. Which of the following series is geometric?

A. $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} + \ldots$

B. $x + 2x + 3x + \ldots$

C. $x^2 + (x+1)^2 + (x+2)^2 + \ldots$

D. $(x+1) + (x+1)^2 + (x+1)^3 + \ldots$

26. In a geometric sequence, if $t_6 = 162$ and $t_3 = 6$, then t_1 is

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. 3

27. $\sum_{k=2}^{18} (4k + 3)$ is equal to

A. 680
B. 688
C. 731
D. 774

28. $\lim_{n \to \infty} \left(\frac{4n - 6n^2 - 5}{2n^2 - 5n} \right)$ is

A. -4
B. -3
C. 2
D. 3

29. The limit of the sequence $-7, 7, -7, 7, -7, \ldots, 7(-1)^n, \ldots$ is

A. non-existent
B. 0
C. 7
D. -7

30. If the infinite geometric series $(x - 2)^1 + (x - 2)^2 + (x - 2)^3 + \ldots$ is convergent, then x satisfies

A. $x > 1$
B. $x < 3$
C. $1 \leqslant x \leqslant 3$
D. $1 < x < 3$

31. A pendulum swings through an arc of 6 cm and the length of each succeeding arc is $\frac{5}{6}$ of the preceding one. The total distance the pendulum will travel in coming to rest is

A. 36 cm
B. 24 cm
C. 18 cm
D. 12 cm

32. The range of the data at the right is

18	20	25	28	14
17	24	28	26	28
14	28	29	27	25
15	28	22	24	20

A. 14
B. 15
C. 25
D. 28

33. The standard deviation for a set of N scores is S. How is the standard deviation affected when a constant, K, is subtracted from each score in the set?

A. It becomes $S - K$.

B. It becomes $S - \frac{K}{N}$.

C. It does not change.

D. It decreases by $\sqrt{K}$.

34. In a normal distribution, the measures of central tendency that must be equal are

A. median and mode but not mean
B. mean and mode but not median
C. mean and median but not mode
D. mean, median, and mode

35. If 3, 7, 11, 15 are a population (n), the standard deviation is

A. 4.0
B. 4.5
C. 6.5
D. 9.0

36. The heights of 1700 students were analyzed. If the heights were normally distributed about a mean of 173 cm with a standard deviation of 13.5 cm, the expected number of students who are taller than 186.5 cm is

A. 1428
B. 1122
C. 578
D. 272

37. The lengths of six pieces of board are as follows: 4 m, 2 m, 5 m, 9 m, 10 m, and 12 m. The number of pieces within one standard deviation of the mean is

A. 5
B. 4
C. 3
D. 2

38. According to the table at the right, the probability of a 20-year-old living to 50 is

A. 0.92
B. 0.95
C. 0.88
D. 0.78

Age	Number living at each age
0	1000
10	968
20	959
30	948
40	925
50	883
60	779

39. Refer to the table at the right. What is the probability (to the nearest 0.01) that a tube will last longer than 732.5 h?

A. 0.45
B. 0.25
C. 0.75
D. 0.32

Lifetime of 130 Radio Tubes	
Lifetime (h)	Frequency
400.5 - 475.5	14
475.5 - 550.5	18
550.5 - 625.5	26
625.5 - 700.5	30
700.5 - 775.5	22
775.5 - 850.5	14
850.5 - 925.5	6

40. Solve for x: $9^{4x-3} = 27^{2x+8}$

A. $x = 15$

B. $x = 9$

C. $x = \frac{11}{2}$

D. $x = \frac{5}{2}$

41. Which of the following is equivalent to $a = \log_c(b)$?

A. $b = a^c$

B. $b = c^a$

C. $a = b^c$

D. $a = c^b$

42. If $x = \log_2(32)$, then x is equal to

A. 5

B. 16

C. 2^{32}

D. 32^2

43. The value of x in the equation $3^{x-1} = 26$ is

A. 1.97
B. 2.97
C. 3.97
D. 0.96

44. $\frac{\log(r)}{t} + \frac{\log(d)}{t}$ is equal to

A. $\log (rd)^t$

B. $\log \left(\sqrt[t]{rd}\right)$

C. $\log \left(\sqrt[t]{(r+d)}\right)$

D. $\log (r+d)^t$

45. The time in years it takes for an investment to increase its value to a specified amount is given by

$$\text{time} = \frac{9}{\log(2)} \log \left(\frac{\text{final amount}}{\text{initial investment}} \right)$$

How many years will it take for a \$600 investment to increase to \$6000?

A. 40
B. 30
C. 13.5
D. 4.3

46. The polynomial $P(x) = 4x^3 - 3x^4 + \frac{5}{2}x^2 - 3$ represents

A. an integral polynomial of degree 3
B. a rational polynomial of degree 3
C. an integral polynomial of degree 4
D. a rational polynomial of degree 4

47. When $2x^3 - 5x^2 - 9x + 9$ is divided by $x - 3$, the quotient and remainder are

A. $2x^2 + x - 6$ and -9

B. $2x^2 + x - 6$ and 27

C. $2x^2 - x - 12$ and -9

D. $2x^2 - 11x + 24$ and -63

48. The synthetic division $\begin{array}{r|rrr} -1 & 1 & 0 & 2 & -5 \\ & & -1 & -1 & -3 \\ \hline & 1 & 1 & 3 & -2 \end{array}$ results from

A. $(x^3 + 2x^2 - 5x) \div (x + 1)$

B. $(x^3 + 2x^2 - 5x) \div (x - 1)$

C. $(x^3 + 2x - 5) \div (x - 1)$

D. $(x^3 + 2x - 5) \div (x + 1)$

49. The integral values at which the graph of $y = x^3 - 12x + 16$ intersects the x-axis are

A. $-2, -4$
B. $-2, 4$
C. $2, -4$
D. $2, 4$

50. If $x^3 - 7x^2 + kx + 3$ is divided by $x - 3$ and the remainder is -18, then k is

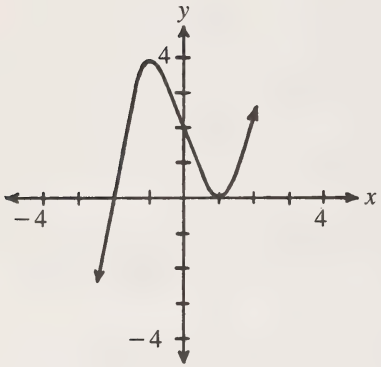
A. -17
B. -5
C. 5
D. 17

51. The complete factorization of $x^4 - 5x^3 + 5x^2 + 5x - 6$ is

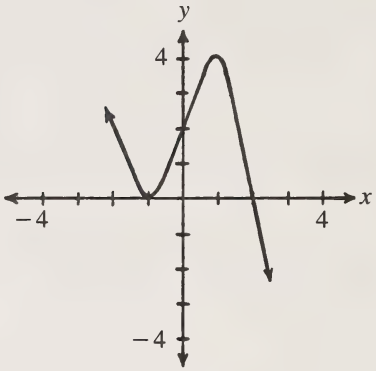
A. $(x + 1)(x - 1)(x - 2)(x + 3)$
B. $(x + 1)(x - 1)(x + 2)(x - 3)$
C. $(x + 1)(x - 1)(x - 2)(x - 3)$
D. $(x + 1)(x - 1)(x + 2)(x + 3)$

52. If $P(x) = (x - 1)^2(x + 2)$, then the sketch that best illustrates the graph of $y = P(x)$ is

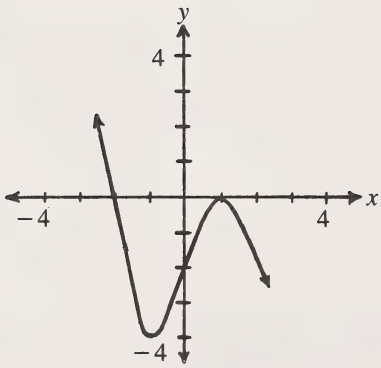
A.



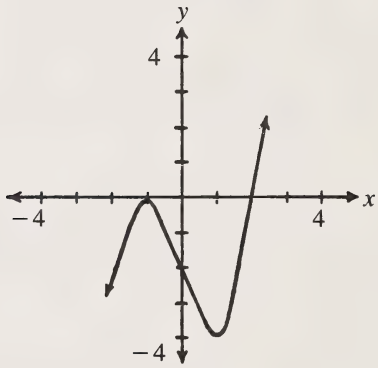
B.



C.



D.



YOU HAVE NOW COMPLETED THE MULTIPLE-CHOICE SECTION OF THE EXAMINATION. PLEASE PROCEED TO THE NEXT PAGE AND ANSWER THE WRITTEN-RESPONSE QUESTIONS IN PART B.

PART B

INSTRUCTIONS

One mark will be given for each correct answer. The remainder of the marks assigned to each question will be given for correct method and/or appropriate diagram.

The VALUE assigned to each question is indicated to the left of the space provided to answer the question.

Place your final answer in the space provided. Show calculations and units used.

TOTAL MARKS: 13

START PART B IMMEDIATELY

(USE FOR ROUGH WORK ONLY)

1. The angle of elevation to the top of a building is 30°. When you move 25 m closer to the building, the angle of elevation becomes 45°. How tall is the building? (Answer to the nearest metre.)

(3 marks)

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2. The sun is at one focus of the elliptical orbit of a comet. The comet's farthest point from the centre of the sun is 256 million km, and its closest point is 16 million km. Determine the length of the minor axis of this elliptical orbit.

(3 marks)

(USE FOR ROUGH WORK ONLY)

3. John deposits equal semi-annual instalments in a fund that bears interest at 10% per annum compounded semi-annually. What sum must be deposited so that, immediately after the 4th deposit has been made, the balance in the account is $650?

(3 marks)

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4. On a comprehensive mathematics test, the mean was 55 and the standard deviation was 5. If you scored 64 on the test, calculate your *z*-score.

(2 marks)

(USE FOR ROUGH WORK ONLY)

5. Solve the equation $(3.28)^x = 0.719$ for x. Round off the answer to 3 significant digits.

(2 marks)

YOU HAVE NOW COMPLETED THE EXAMINATION. IF YOU HAVE TIME, YOU MAY WISH TO GO BACK AND CHECK YOUR ANSWERS.

FOR DEPARTMENT USE ONLY

M1 M2 M3

FOR DEPARTMENT USE ONLY

(LAST NAME) (FIRST NAME)

NAME:

Y M D

DATE OF BIRTH: SEX:

SCHOOL CODE: SCHOOL:

CITY:

NAME OF EXAMINATION:

SIGNATURE:

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